

Combined Effects of Radiation and Chemical Reaction on Free Convection Hydromagnetic Flow Past an Impulsively Started Vertical Plate with Variable Concentration in Porous Medium

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Abstract—In this paper an investigation has been made to study the effect of thermal radiation on free convection hydromagnetic flow past a vertical moving plate with variable concentration under the assumption of first order Chemical reaction in porous medium. The governing equations are reduced to non-dimensional forms and they are solved in closed form by Laplace Transform Technique. Computations have been carried out with the subroutine of complementary error functions. In order to know the effect of different physical parameters involved in the process viz. thermal Grashof number (Gr), mass Grashof number (G_m), Schmidt number (Sc), Radiation parameter (F), Chemical reaction parameter (R), Prandtl number (P_r) and time (t) on the physical flow field, computations are carried out in FORTRAN for concentration, temperature and vertical velocity and they are presented graphically. It is observed that Chemical reaction parameter, radiation parameter and magnetic parameter have significant effect on flow pattern.

Keywords: Free Convection, Moving vertical plate, Magneto-hydrodynamics, Radiation, Chemical reaction, Porous medium.

1. INTRODUCTION

MHD flow has application in metrology, solar physics and in motion of earth core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Magneto-convection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Heat transfer with radiation effects also has mathematical as well as physical importance and many researchers found interest it as a subject of investigation. Mass transfer with chemical reaction is another most commonly encountered circumstance in chemical industry as well as in physical and biological sciences. There are many situations where convection heat transfer

Phenomena are accompanied by mass transfer as well as radiation also. When mass transfer takes place in a fluid at

rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer processes are similar in nature. Studies in porous medium are other important areas in heat transfer processes. Due to these important industrial and engineering applications, magneto-convection with radiation and chemical reaction has been gaining considerable attention amongst researchers. Hence a study combining all these aspects will surely enhance the already developed areas further for more complex studies.

Illingworth [12] first studied the unsteady laminar flow near an infinite flat plate and Siegel [21] first reported the transient free convection flow past an isothermal vertical plate. Soundalgekar [22] first studied the Stokes problem for flow past an impulsively started infinite vertical plate. Simultaneous heat and mass transfer in laminar free convection boundary layer flows over surfaces was studied by Gebhart *et. al.* [8]. Effects of mass transfer and free convection currents on the flow past an impulsively started vertical isothermal plate was studied by Soundalgekar [23]. Gebhart & Pera [9] studied the nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. And Das *et. al.* [5] studied the effects of mass transfer on flow past an impulsively started vertical infinite plate with constant heat flux and chemical reaction.

Earlier studies on convective heat transfer in porous medium were concerned in most of the cases with the steady state conditions only (ref. Nield & Bejan [17]). But, wide range of engineering applications led to an increasing interest amongst researchers to study the unsteady flows also. Porous medium also gets much attention among researchers because of wide applications. Free convection mass diffusion in porous

medium was studied by Jang & Ni [13], Pop & Herwig [18] and Lai & Kulacki [14]. Ramachandra and Reddy [19] investigated the radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate in a porous medium with viscous dissipation. Choudhary & Jain [2] investigated MHD free convection with mass diffusion process along a vertical plate. On the other hand, England and Emery [7] considered the thermal radiation effects on the laminar free convection boundary layer of an absorbing gas bounded by a vertical stationary plate. Soundalgekar and Takhar [24] studied the effect of radiation on free convection flow past a semi infinite vertical plate. Das *et. al.*[3] studied the effects of mass transfer on flow past an impulsively started vertical infinite plate with constant heat flux and chemical reaction. They [4] also considered the case of radiation effects on flow past an impulsively started vertical infinite plate. Hossain and Takhar [11] studied the radiation effect on mixed convection along a vertical plate with uniform surface temperature. Raptis and Perdikis [20] considered the radiation and free convection flow past a moving plate. Muthucumaraswamy *et. al.* [16] studied the radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Mazumdar [15] considered the MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation. Recently; Deka and Neog [6] considered the combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium.

Although many authors studied mass transfer with or without chemical reaction in flow past vertical plates under different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer in the presence of radiation and chemical reaction through vertical plate with periodic temperature has not been found in literature and hence we are motivated to undertake this study.

2. MATHEMATICAL ANALYSIS

We have considered here an unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate in porous medium. To visualize the flow pattern a Cartesian co-ordinate system is considered where x' -axis is taken along the infinite vertical plate, y' -axis is normal to the plate and fluid fills the region $y' \geq 0$. Initially, the fluid and the plate are kept at the same constant temperature T'_∞ and species concentration C'_∞ . At time $t' > 0$, the plate is given an impulsive motion in its own plane with a velocity U_0 . At the same time the plate temperature is raised to T'_w and concentration is raised linearly and a magnetic field of uniform strength B_0 is applied normal to the plate. It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so the Soret and Dufour effects are negligible.

As the plate is infinite in extent so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only as a result the motion becomes one dimensional with only non-zero vertical velocity component u' , varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied.

Under the above assumptions and following Boussinesq approximation, the unsteady flow field is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{\kappa} u' \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

Along with the following initial and boundary conditions:

$$\left. \begin{aligned} u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all } y' \text{ and } t' \leq 0 \\ u' = U_0, \quad T' = T'_w + (T'_w - T'_\infty) At', \quad C' = C'_w \quad \text{at } y' = 0 \\ u' \rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \end{aligned} \right\}, t' > 0 \tag{4}$$

Where, $A = \frac{u_0^2}{\nu}$.

Now to reduce the above equations in non-dimensional form we introduce the following non-dimensional quantities.

$$\left. \begin{aligned} u = \frac{u'}{U_0}, \quad t = \frac{t' U_0^2}{\nu}, \quad y = \frac{y' U_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \\ \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U_0^3}, \\ Gm = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U_0^3}, \quad Pr = \frac{\mu C_p}{\kappa}, \\ R = \frac{\nu K_1}{U_0^2}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho U_0^2} \end{aligned} \right\} \tag{5}$$

Thus with the help of these non-dimensional quantities, equations (1), (2) and (3) reduce to:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - M'u \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{F}{Pr} \theta \tag{7}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - R\phi \tag{8}$$

And the initial and boundary conditions are as follows:

$$\left. \begin{aligned} u = 0, \quad \theta = 0, \quad \phi = 0, \text{ for all } y \text{ and } t \leq 0 \\ u = 1, \quad \theta = 1, \quad \phi = t \text{ at } y = 0 \\ u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\}, t > 0 \tag{9}$$

Solutions of the equations (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained with the help of Abramowitz and Stegun [16] and Hetnarski's [17] algorithm. They are obtained as follows:

$$\theta(y,t) = \frac{1}{2} \left\{ \begin{aligned} & e^{-y\sqrt{aPr}} \operatorname{erfc} \left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at} \right) + \\ & e^{y\sqrt{ScR}} \operatorname{erfc} \left(\frac{y\sqrt{Pr}}{2\sqrt{t}} + \sqrt{at} \right) \end{aligned} \right\} \tag{10}$$

$$\phi(y,t) = \frac{1}{2} \left\{ \begin{aligned} & e^{-y\sqrt{ScR}} \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt} \right) + \\ & e^{y\sqrt{ScR}} \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Rt} \right) \end{aligned} \right\} \tag{11}$$

$$\begin{aligned} u(y,t) = & G_3 \left[\frac{1}{2} \left\{ e^{-y\sqrt{M'}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{M'/t} \right) + cc \right\} + \right. \\ & \frac{G_1}{b} \left[\frac{e^{bt}}{2} \left\{ e^{-y\sqrt{M'+b}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{(M'+b)t} \right) + cc \right\} - \right. \\ & \left. \frac{e^{bt}}{2} \left\{ e^{-y\sqrt{Pr}\sqrt{(a+b)}} \operatorname{erfc} \left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{(a+b)t} \right) + cc \right\} + \right. \\ & \left. \frac{1}{2} \left\{ e^{-y\sqrt{Pra}} \operatorname{erfc} \left(\frac{y\sqrt{Pr}}{2\sqrt{t}} - \sqrt{at} \right) + cc \right\} + \right. \\ & \frac{G_2}{d^2} \left[\frac{e^{dt}}{2} \left\{ e^{-y\sqrt{M'+d}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{(M'+d)t} \right) + cc \right\} - \right. \\ & \left. \frac{e^{dt}}{2} \left\{ e^{-y\sqrt{Sc}\sqrt{R+d}} \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(R+d)t} \right) + cc \right\} + \right. \\ & \left. \frac{1}{2} \left\{ e^{-y\sqrt{ScR}} \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt} \right) + cc \right\} \right] - \\ & \frac{G_2}{d} \left[\left(\frac{t}{2} \left\{ e^{-y\sqrt{M'}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{M'/t} \right) + cc \right\} - \right. \right. \\ & \left. \left. \frac{y}{4\sqrt{M'}} \left\{ e^{-y\sqrt{M'}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{M'/t} \right) - cc \right\} - \right. \right. \\ & \left. \left. \frac{t}{2} \left\{ e^{-y\sqrt{ScR}} \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt} \right) + cc \right\} - \right. \right. \\ & \left. \left. \frac{y\sqrt{Sc}}{4\sqrt{R}} \left\{ e^{-y\sqrt{ScR}} \operatorname{erfc} \left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt} \right) - cc \right\} \right] \end{aligned} \tag{12}$$

Here, the following symbols are used in the above solutions:

$$\left. \begin{aligned} a = \frac{F}{Pr}, M = M + \frac{1}{K}, b = \frac{M - aPr}{Pr - 1}, d = \frac{RSe - M}{1 - Sc}, \\ G_1 = \frac{Gr}{Pr - 1}, G_2 = \frac{Gm}{Sc - 1}, G_3 = 1 - \frac{G_1}{b} \frac{G_2}{d^2} \end{aligned} \right\} \tag{13}$$

3. RESULTS AND DISCUSSION

The numerical values of the velocity, temperature and concentration fields are computed for different parameters like magnetic field parameter, radiation parameter, chemical reaction parameter, Schmidt number, Prandtl number, thermal Grashof number and mass Grashof number and phase angle and they are presented graphically in figure.

Fig. 1 represents the temperature profiles showing the effect of Pr and F for different values of Pr (0.71, 7) and F (0.5, 5). From this Fig. it is clear that temperature decreases with the increase of Pr and F . In figures 2(a) and 2(b) concentration profiles are presented for different values of Sc (0.6, 3.5) and R (2, 5). It is observed that increase of Schmidt number and chemical reaction parameter lead to decrease in concentration.

Velocity profiles for different values of parameters are shown in figures 3-6. Effect of Gr (10, 20) and Gm (2, 5) are presented in Fig. 3 for some fixed values of other parameters. It is observed that increase of Gr and Gm lead to increase of velocity. Influence of M (0.5, 1) and K (1, 10) are shown in Fig. 4 for some fixed values of the other parameters and seen from this Fig. that increase of M leads to decrease of velocity whereas increase of K leads to increase of velocity. In Fig. 5 velocity profiles are presented for different values of Sc (0.2, 0.6) and R (2, 5). It is clear from these figures that velocity decreases with the increase of Sc and R . Fig. 6 represents velocity profiles for different values of Pr (0.71, 7) and F (0.05, 0.5, 5). We have presented this Fig. for two different mediums whereas in the other figures we have presented graphs for only one medium. An interesting observation made from this Fig. is that when $Pr=0.71$ velocity shoots abruptly above the normal level than for $Pr=7$. Which signifies that in lighter medium velocity is more than thicker medium. Thus increase in Pr leads to decrease in velocity. Also increase in F increases velocity.

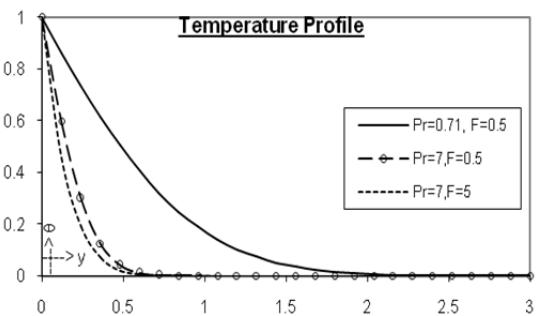


Fig. 1: Temperature Profile showing the effect of Pr and F .

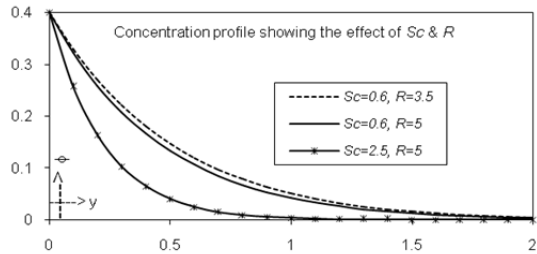


Fig. 2(a) Concentration Profile

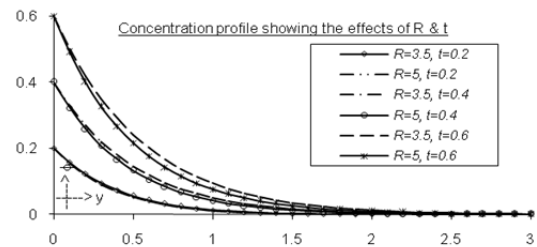


Fig. 2(b) Time dependent Concentration Profile

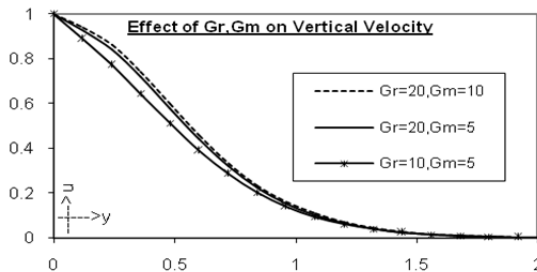


Fig. 3 Velocity Profile showing the effect of Gr & Gm

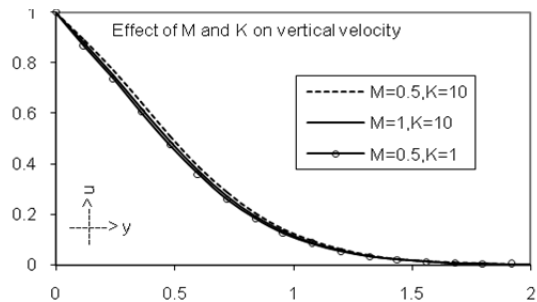


Fig. 4 Velocity Profile showing the effect of M, K

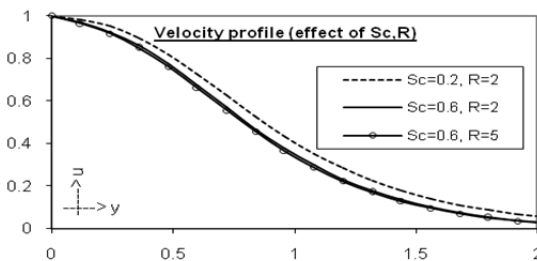


Fig. 5 Velocity Profile showing the effect of Sc, R

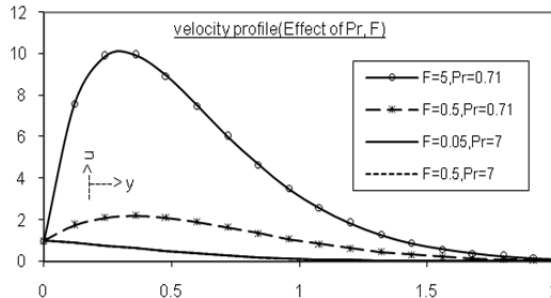


Fig. 6: Velocity Profile showing the effect of Pr, F

4. CONCLUSIONS

An exact analysis in closed form is performed to study the influence of chemically reacting hydromagnetic flow past a vertical moving plate in porous medium. Solutions are obtained by Laplace transform technique. Some of the important conclusions of the study are as follows:

- Temperature decreases with the increase of Pr and F .
- Concentration decreases as Sc and R increase.
- Velocity increases with increasing Gr, Gm and K and with decreasing M and F .
- Also increase in Sc and R lead to decrease in velocity.
- When Pr increases velocity decreases.

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